電気回路演習 II 第 13 回 (平成 20 年 7 月 11 日 (金))

演習

1. 以下の関数をラプラス変換しなさい

   (a) $f_1(t) = t^2$ (b) $f_2(t) = \begin{cases} 0 & (0 \le t < a) \\ 1 & (a \le t) \end{cases}$

2. 問 1 の結果と推移定理を用いて以下の関数をラプラス変換しなさい

   $f_3(t) = t^2 e^{-3t}$

3. 以下の関数のラプラス逆変換を求めなさい

   (a) $F_4(s) = \dfrac{3s+1}{s^2+6s+8}$ (b) $F_5(s) = \dfrac{s+12}{s^2+16}$ (c) $F_6(s) = \dfrac{s+3}{(s+2)^2(s+1)}$

演習解答

1. (a) $f_1(t) = t^2$

$$\begin{aligned} F_1(s) = \mathcal{L}\{f_1(t)\} &= \int_0^\infty t^2 e^{-st}dt = \left[t^2\frac{e^{-st}}{-s}\right]_0^\infty + \int_0^\infty 2t\frac{e^{-st}}{s}dt \\ &= 0 + \left[2t\frac{e^{-st}}{-s^2}\right]_0^\infty + \int_0^\infty 2\frac{e^{-st}}{s^2}dt = 0 + 0 + \left[2\frac{e^{-st}}{-s^3}\right]_0^\infty = \frac{2}{s^3} \end{aligned}$$

   (b) $f_2(t) = \begin{cases} 0 & (0 \le t < a) \\ 1 & (a \le t) \end{cases}$

$$F_2(s) = \mathcal{L}\{f_2(t)\} = \int_0^a 0\cdot e^{-st}dt + \int_a^\infty 1\cdot e^{-st}dt = \left[-\frac{e^{-st}}{s}\right]_a^\infty = \frac{e^{-sa}}{s}$$

2. (a) 問 1 の結果と推移定理を用いると

$$F_3(s) = \mathcal{L}\left\{t^2 e^{-3t}\right\} = \mathcal{L}\left\{f_1(t)e^{-3t}\right\} = F_1(s+3) = \frac{2}{(s+3)^3}$$

3. (a) $F_4(s) = \dfrac{3s+2}{(s+4)(s+2)}$

$$\begin{aligned} f_4(t) &= (s+4)F_4(s)e^{st}\Big|_{s=-4} + (s+2)F_4(s)e^{st}\Big|_{s=-2} \\ &= \frac{3s+2}{s+2}e^{st}\Big|_{s=-4} + \frac{3s+2}{s+4}e^{st}\Big|_{s=-2} = 5e^{-4t} - 2e^{-2t} \end{aligned}$$

   (b) $F_5(s) = \dfrac{s+3\cdot 4}{s^2+4^2} = \dfrac{s}{s^2+4^2} + 3\cdot\dfrac{4}{s^2+4^2}$

   ラプラス変換表より

$$f_5(t) = \mathcal{L}^{-1}\{F_5(s)\} = \cos 4t + 3\sin 4t$$

   (c) 与式を以下のように変形する

$$\begin{aligned} \frac{s+3}{(s+2)^2(s+1)} &= \frac{K_1}{(s+2)^2} + \frac{K_2}{s+2} + \frac{K_3}{s+1} = \frac{K_1(s+1) + K_2(s+2)(s+1) + K_3(s+2)^2}{(s+2)^2(s+1)} \\ &= \frac{(K_2+K_3)s^2 + (K_1+3K_2+4K_3)s + (K_1+2K_2+4K_3)}{(s+2)^2(s+1)} \end{aligned}$$

   係数比較により

$$\begin{cases} K_2 + K_3 = 0 \\ K_1 + 3K_2 + 4K_3 = 1 \\ K_1 + 2K_2 + 4K_3 = 3 \end{cases} \quad \rightarrow \quad K_1 = -1,\ K_2 = -2,\ K_3 = 2$$

   よって

$$f_6(t) = \mathcal{L}^{-1}\{F_6(s)\} = \mathcal{L}^{-1}\left\{\frac{-1}{(s+2)^2} + \frac{-2}{s+2} + \frac{2}{s+1}\right\} = -te^{-2t} - 2e^{-2t} + 2e^{-t}$$

小テスト

1. 以下の関数をラプラス変換しなさい

   (a) $f_1(t) = e^{-at}$　(b) $f_2(t) = u(t-T)$　(b) $f_3(t) = \begin{cases} 1 & (0 \le t < a) \\ 0 & (a \le t) \end{cases}$

2. 以下の関数のラプラス逆変換を求めなさい

   (a) $F_4(s) = \dfrac{10s+6}{s^2+10s+21}$　(b) $F_5(s) = \dfrac{2s+3}{s^2+36}$

小テスト解答

1. (a) $\mathcal{L}\{f_1(t)\} = \int_0^\infty e^{-at}e^{-st}dt = \int_0^\infty e^{-(s+a)t}dt = \left[\dfrac{e^{-(s+a)t}}{-(s+a)}\right]_0^\infty = \dfrac{1}{s+a}$

   (b) $\mathcal{L}\{f_2(t)\} = \int_0^\infty u(t-T)e^{-st}dt = \int_T^\infty e^{-st}dt = \left[\dfrac{e^{-st}}{-s}\right]_T^\infty = \dfrac{e^{-sT}}{s}$

   (c) $\mathcal{L}\{f_3(t)\} = \int_0^a 1 \cdot e^{-st}dt + \int_a^\infty 0 \cdot e^{-st}dt = \left[-\dfrac{e^{-st}}{s}\right]_0^a = \dfrac{1-e^{-as}}{s}$

2. (a) $F_4(s) = \dfrac{10s+6}{(s+3)(s+7)}$

$$\begin{aligned} f_4(t) &= (s+3)F_4(s)e^{st}\Big|_{s=-3} + (s+7)F_4(s)e^{st}\Big|_{s=-7} \\ &= \frac{10s+6}{s+7}e^{st}\Bigg|_{s=-3} + \frac{10s+6}{s+3}e^{st}\Bigg|_{s=-7} = -6e^{-3t} + 16e^{-7t} \end{aligned}$$

   (b) $F_5(s) = \dfrac{2s + \frac{1}{2}\cdot 6}{s^2+6^2} = 2 \cdot \dfrac{s}{s^2+6^2} + \dfrac{1}{2} \cdot \dfrac{6}{s^2+6^2}$

   ラプラス変換表より

$$f_5(t) = \mathcal{L}^{-1}\{F_5(s)\} = 2\cos 6t + \frac{1}{2}\sin 6t$$