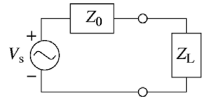
最大電力伝送定理

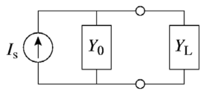
負荷インピーダンスと電源の内部インピーダンスが複素共役の関係(共役整合)にあるとき、負荷に供給される電力は最大になる



$$P = \frac{R_L |V_s|^2}{(R_0 + R_L)^2 + (X_0 + X_L)^2}$$

$$Z_{L,opt} = Z_0^* \quad \Rightarrow \quad P_{\max} = \frac{|V_s|^2}{4R_0}$$

$$\begin{pmatrix} R_{L,opt} = R_0 \\ X_{L,opt} = -X_0 \end{pmatrix}$$



$$P = \frac{G_L |I_s|^2}{(G_0 + G_L)^2 + (B_0 + B_L)^2}$$

$$Y_{L,opt} = Y_0^* \quad \Rightarrow \quad P_{\max} = \frac{|I_s|^2}{4G_0}$$

$$\begin{pmatrix} G_{L,opt} = G_0 \\ B_{L,opt} = -B_0 \end{pmatrix}$$

最大電力伝送定理の証明

$$P = \frac{R_L |V_s|^2}{(R_0 + R_L)^2 + (X_0 + X_L)^2}$$

⬇ 分母を最小化する $X_L = -X_0$

$$P = \frac{R_L |V_s|^2}{(R_0 + R_L)^2} = \frac{|V_s|^2}{R_0\left(\sqrt{\frac{R_0}{R_L}} + \sqrt{\frac{R_L}{R_0}}\right)^2} = \frac{|V_s|^2}{R_0\left(x + \frac{1}{x}\right)^2} \qquad \left(x = \sqrt{\frac{R_0}{R_L}}\right)$$

⬇ 分母を最小化する $x = \frac{1}{x} = 1$

$R_L = R_0$

$$P = \frac{|V_s|^2}{4R_0}$$

x が正の実数のとき

$$x + \frac{1}{x} \geq 2$$

等号は $x = 1/x = 1$ のとき

R_L のみが変化できるとき

$$\frac{\partial P}{\partial R_L} = \frac{\left\{(R_0 + R_L)^2 + (X_0 + X_L)^2\right\} - 2R_L(R_0 + R_L)}{\left\{(R_0 + R_L)^2 + (X_0 + X_L)^2\right\}^2} |V_s|^2$$

$$= \frac{\left\{(R_0 + R_L)(R_0 - R_L) + (X_0 + X_L)^2\right\}}{\left\{(R_0 + R_L)^2 + (X_0 + X_L)^2\right\}^2} |V_s|^2$$

$$= \frac{\left\{R_0^2 - R_L^2 + (X_0 + X_L)^2\right\}}{\left\{(R_0 + R_L)^2 + (X_0 + X_L)^2\right\}^2} |V_s|^2 = 0 \quad \Rightarrow \quad R_L = \sqrt{R_0^2 + (X_0 + X_L)^2}$$

微分による最大・最小の計算

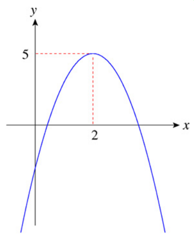
$$y = -2x^2 + 8x - 3 \longrightarrow y = -2(x-2)^2 + 5$$

⬇ $\frac{\partial y}{\partial x} = 0$ で極値

$$\frac{\partial y}{\partial x} = -4x + 8 = 0$$

$$\therefore \quad x = 2$$

$$y\big|_{x=2} = -2 \cdot 2^2 + 8 \cdot 2 - 3 = 5$$



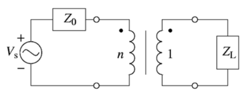
X_L/R_L 一定で大きさを変化させるとき

$$P = \frac{n^2 R_L |V_s|^2}{(R_0 + n^2 R_L)^2 + (X_0 + n^2 X_L)^2} = \frac{R_L |V_s|^2}{\left(\frac{R_0}{n} + nR_L\right)^2 + \left(\frac{X_0}{n} + nX_L\right)^2}$$

⬇ 分母を最小化する

$\vec{A} = (R_0, X_0), \quad \vec{B} = (R_L, X_L)$ と考えると

$$\left(\frac{R_0}{n} + nR_L\right)^2 + \left(\frac{X_0}{n} + nX_L\right)^2 = \left|\frac{\vec{A}}{n} + n\vec{B}\right|^2$$



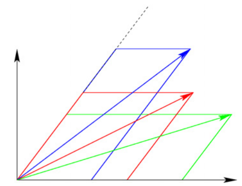
一方 $\frac{\vec{A}}{n} \times n\vec{B} = \vec{A} \times \vec{B} \longrightarrow$ 面積一定

よって面積一定の下で対角線の長さを最小にする

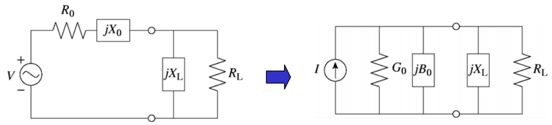
$$\left|\frac{\vec{A}}{n}\right| = \left|n\vec{B}\right| \longrightarrow \left|\vec{A}\right| = n^2\left|\vec{B}\right|$$

⬇

$$|Z_0| = n^2 |Z_L|$$



例) R_L のみが可変であるとき、負荷への供給電力を最大にする



$$G_0 = \frac{R_0}{R_0^2 + X_0^2}, \quad B_0 = \frac{-X_0}{R_0^2 + X_0^2}$$

$$G_{L,opt} = \sqrt{G_0^2 + \left(B_0 + B_L\right)^2} = \sqrt{\left(\frac{R_0}{R_0^2 + X_0^2}\right)^2 + \left(\frac{-X_0}{R_0^2 + X_0^2} - \frac{1}{X_L}\right)^2}$$

$$= \sqrt{\left(\frac{R_0}{R_0^2 + X_0^2}\right)^2 + \left(\frac{R_0^2 + X_0^2 + X_0 X_L}{\left(R_0^2 + X_0^2\right)X_L}\right)^2} = \sqrt{\frac{R_0^2 X_L^2 + \left(R_0^2 + X_0^2\right)^2 + 2X_0 X_L\left(R_0^2 + X_0^2\right) + X_0^2 X_L^2}{\left(R_0^2 + X_0^2\right)^2 X_L^2}}$$

$$= \sqrt{\frac{\left(R_0^2 + X_0^2\right)\left\{R_0^2 + X_0^2 + 2X_0 X_L + X_0^2 X_L^2\right\}}{\left(R_0^2 + X_0^2\right)^2 X_L^2}}$$

$$= \frac{1}{X_L}\sqrt{\frac{R_0^2 + \left(X_0 + X_L\right)^2}{\left(R_0^2 + X_0^2\right)}} \longrightarrow R_{L,opt} = X_L\sqrt{\frac{R_0^2 + X_0^2}{R_0^2 + \left(X_0 + X_L\right)^2}}$$