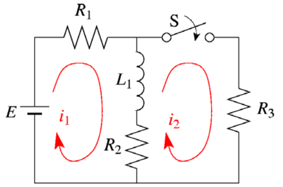
より複雑な回路の過渡現象



$E = 60\,\mathrm{V},\ L = 5\,\mathrm{H}$

$R_1 = R_2 = 10\,\Omega,\ \ R_3 = 5\,\Omega$

$t=0$で　$i_1 = \dfrac{E}{R_1+R_2} = \dfrac{60}{20} = 3\,\mathrm{A},\ i_2 = 0\,\mathrm{A}$

$t \geq 0$で　回路方程式は

$$\begin{cases} L_1 \dfrac{d(i_1-i_2)}{dt} + R_1 i_1 + R_2(i_1-i_2) = E \\ L_1 \dfrac{d(i_2-i_1)}{dt} + R_2(i_2-i_1) + R_3 i_2 = 0 \end{cases}$$

$$\begin{cases} \left(L_1\dfrac{d}{dt}+R_1+R_2\right)i_1 - \left(L_1\dfrac{d}{dt}+R_2\right)i_2 = E \\ -\left(L_1\dfrac{d}{dt}+R_2\right)i_1 + \left(L_1\dfrac{d}{dt}+R_2+R_3\right)i_2 = 0 \end{cases}$$

与えられた数値を代入すると

$$\begin{cases} \left(5\dfrac{d}{dt}+20\right)i_1 - \left(5\dfrac{d}{dt}+10\right)i_2 = 60 \\ -\left(5\dfrac{d}{dt}+10\right)i_1 + \left(5\dfrac{d}{dt}+15\right)i_1 = 0 \end{cases} \Rightarrow \begin{cases} \left(\dfrac{d}{dt}+4\right)i_1 - \left(\dfrac{d}{dt}+2\right)i_2 = 12 \\ -\left(\dfrac{d}{dt}+2\right)i_1 + \left(\dfrac{d}{dt}+3\right)i_1 = 0 \end{cases}$$

ラプラス変換すると

$$\begin{cases} \{sI_1(s)-i_1(0)+4I_1(s)\} - \{sI_2(s)-i_2(0)+10I_2(s)\} = \dfrac{12}{s} \\ -\{sI_1(s)-i_1(0)+10I_1(s)\} + \{sI_2(s)-i_2(0)+3I_2(s)\} = 0 \end{cases}$$

行列の形でまとめると

$$\begin{bmatrix} s+4 & -(s+2) \\ -(s+2) & s+3 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12}{s}+i_1(0)-i_2(0) \\ i_2(0)-i_1(0) \end{bmatrix} = \begin{bmatrix} \dfrac{12}{s}+3 \\ -3 \end{bmatrix}$$

左辺行列の行列式は

$$\Delta = (s+4)(s+3)-(s+2)^2 = (s^2+7s+12)-(s^2+4s+4) = 3s+8$$

Cramer の公式より

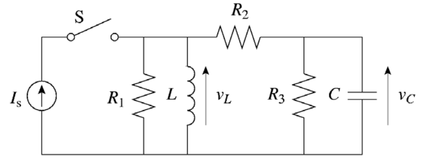
$$I_1(s) = \frac{\begin{vmatrix} \dfrac{12}{s}+3 & -(s+2) \\ -3 & s+3 \end{vmatrix}}{3s+8} = \frac{\left(3s+21+\dfrac{36}{s}\right)-(3s+6)}{3s+8} = \frac{15s+36}{s(3s+8)}$$

$$I_2(s) = \frac{\begin{vmatrix} s+4 & \dfrac{12}{s}+3 \\ -(s+2) & -3 \end{vmatrix}}{3s+8} = \frac{(-3s-12)-\left(-3s-18-\dfrac{24}{s}\right)}{3s+8} = \frac{6s+24}{s(3s+8)}$$

ラプラス逆変換すると

$$i_1(t) = sI(s)e^{st}\Big|_{s=0} + \left(s+\frac{8}{3}\right)I(s)e^{st}\Big|_{s=-\frac{8}{3}} = \frac{9}{2}+\frac{1}{2}e^{-\frac{8}{3}t}\ [\mathrm{A}]$$

$$i_2(t) = sI(s)e^{st}\Big|_{s=0} + \left(s+\frac{8}{3}\right)I(s)e^{st}\Big|_{s=-\frac{8}{3}} = 3-e^{-\frac{8}{3}t}\ [\mathrm{A}]$$



$I_s = 2\,\mathrm{A},\ L = 1\,\mathrm{H},\ C = 1\,\mathrm{F}$

$R_1 = R_2 = R_3 = 1\,\Omega$

$t=0$で　$v_L(0) = v_C(0) = 0 \quad i_L(0) = \dfrac{1}{L}\displaystyle\int v_L dt\Big|_{t=0} = \int v_L dt\Big|_{t=0} = 0$

$t \geq 0$で　回路方程式は

$$\begin{cases} \dfrac{1}{R}v_L + \dfrac{1}{L}\displaystyle\int v_L dt + \dfrac{v_L - v_C}{R_2} = I_s u(t) \\ \dfrac{v_C - v_L}{R_2} + \dfrac{1}{R_3}v_C + C\dfrac{dv_C}{dt} = 0 \end{cases}$$

数値を代入して整理すると

$$\begin{cases} 2v_L + \displaystyle\int v_L dt - v_C = 2_s u(t) \\ -v_L + 2v_C + \dfrac{dv_C}{dt} = 0 \end{cases}$$

ラプラス変換すると

$$\begin{cases} 2V_L(s)+\left(\dfrac{1}{s}V_L(s)+\dfrac{1}{s}\displaystyle\int v_L\,dt\Big|_{t=0}\right)-V_C(s)=\dfrac{2}{s} \\ -V_L(s)+2V_C(s)+\left(sV_C(s)-v_C(0)\right)=0 \end{cases}$$

初期条件を考慮して行列の形にまとめると

$$\begin{bmatrix} 2+\dfrac{1}{s} & -1 \\ -1 & s+2 \end{bmatrix}\begin{bmatrix} V_L(s) \\ V_C(s) \end{bmatrix}=\begin{bmatrix} \dfrac{2}{s} \\ 0 \end{bmatrix}$$

上式を特と

$$V_L(s)=\frac{s+2}{(s+1)^2}=\frac{(s+1)+1}{(s+1)^2}=\frac{1}{s+1}+\frac{1}{(s+1)^2}$$

$$V_C(s)=\frac{1}{(s+1)^2}$$

ラプラス逆変換すると

$$v_L(t)=e^{-t}+te^{-t}=(1+t)e^{-t}\ \text{[V]}$$

$$v_C(t)=te^{-t}\ \text{[V]}$$