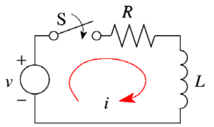
特殊波形に対する過渡現象解析

(1) 単独波形



$$v = Ee^{-bt}u(t)$$

$$i(0)=0$$

回路方程式(閉路方程式)

$$L\frac{di}{dt}+Ri=e^{-bt}u(t)$$

⬇ ラプラス変換

$$L\{sI(s)-i(0)\}+RI(s)=\frac{E}{s+b}$$

⬇ $i(0)=0$

$$I(s)=\frac{E}{L}\left(\frac{1}{(s+b)(s+R/L)}\right)$$

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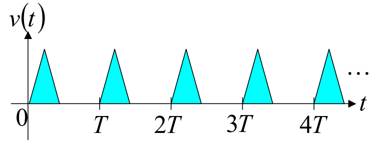
(2) 繰り返し波形



等比級数の和

$$\sum_{n=0}^{N-1} ar^n=\frac{a(1-r^n)}{1-r}$$

⬇ $|r|<1,\quad n\to\infty$

$$\sum_{n=0}^{\infty} ar^n=\frac{a}{1-r}$$

$$v(t)=f(t)+f(t-T)+f(t-2T)+\cdots=\sum_{n=0}^{\infty} f(t-nT)$$

⬇ ラプラス変換

$$V(s)=F(s)+F(s)e^{-Ts}+F(s)e^{-2Ts}+\cdots=F(s)\sum_{n=1}^{\infty}\left(e^{-Ts}\right)^n=\frac{F(s)}{1-e^{-Ts}}$$

$$v(t)=f(t)-f(t-T)+f(t-2T)-\cdots=\sum_{n=0}^{\infty}(-1)^n f(t-nT)$$

⬇ ラプラス変換

$$V(s)=F(s)-F(s)e^{-Ts}+F(s)e^{-2Ts}-\cdots=F(s)\sum_{n=1}^{\infty}\left(-e^{-Ts}\right)^n=\frac{F(s)}{1+e^{-Ts}}$$

$b\neq 0, R/L$のとき

$$i(t)=(s+b)I(s)e^{st}\Big|_{s=-b}+(s+R/L)I(s)e^{st}\Big|_{s=-R/L}$$

$$=\frac{E/L}{s+R/L}e^{st}\Big|_{s=-b}+\frac{E/L}{s+b}e^{st}\Big|_{s=-R/L}$$

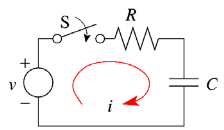
$$=\frac{E}{(R-bL)}e^{-bt}+\frac{E}{(bL-R)}e^{-(R/L)t}$$

$$=\frac{E}{(R-bL)}\left(e^{-bt}-e^{-(R/L)t}\right)$$

$b=R/L$のとき

$$I(s)=\frac{E}{L}\frac{1}{(s+R/L)^2}$$

$$i(t)=\frac{E}{L}te^{-(R/L)t}$$



初期条件

$q(0)=0$

回路方程式(閉路方程式)

$$Ri+\frac{q}{C}=v(t)\ \xrightarrow{\ q(t)=\int i(t)dt\ }\ Ri+\frac{1}{C}\int idt=v(t)$$

⬇ ラプラス変換

$$RI(s)+\frac{1}{C}\left\{\frac{I(s)}{s}+\frac{1}{s}\int idt\Big|_{t=0}\right\}=V(s)=\frac{F(s)}{1-e^{-Ts}}$$

$$q(0)=0$$

$$\left(R+\frac{1}{sC}\right)I(s)=\frac{F(s)}{1-e^{-Ts}}\ \longrightarrow\ I(s)=\frac{s}{R}\frac{F(s)}{\left(s+\frac{1}{RC}\right)\left(1-e^{-Ts}\right)}$$

$$f(t)=\begin{cases}E & 0\le t\le \dfrac{T}{2}\\ 0 & t>\dfrac{T}{2}\end{cases}\quad \text{のとき}$$

$$F(s)=\int_0^{T/2}Ee^{-st}dt=\left[\frac{Ee^{-st}}{-s}\right]_0^{T/2}=\frac{E}{s}\left(1-e^{-\frac{T}{2}s}\right)$$

よって

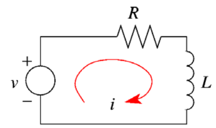
$$I(s)=\frac{s}{R}\frac{E\left(1-e^{-\frac{T}{2}s}\right)}{s\left(s+\frac{1}{RC}\right)\left(1-e^{-Ts}\right)}=\frac{E}{R}\frac{1}{\left(s+\frac{1}{RC}\right)\left(1+e^{-\frac{T}{2}s}\right)}$$

$$g(t)=1+e^{-\frac{T}{2}s}=1+\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{T}{2}s\right)^n=\prod_{i=0}^{\infty}(s+s_i)\quad (\text{s の多項式})$$

$$g(t)=1+e^{-\frac{T}{2}(j\omega)}=\left(e^{\frac{T}{4}(j\omega)}+e^{-\frac{T}{4}(j\omega)}\right)e^{-\frac{T}{4}(j\omega)}=2e^{-\frac{T}{4}(j\omega)}\cos\frac{T}{4}\omega\quad (s=j\omega)$$

$$\omega=\omega_n=\pm\frac{2(2n+1)\pi}{T}\ \text{で}\ g(t)=0\quad (n:\text{正の整数})$$

インパルス応答とその応用



回路方程式(閉路方程式)

$$L\frac{di}{dt}+Ri=\delta(t)$$ ← 単位インパルス入力

ラプラス変換

$$L\{sI(s)-i(0)\}+RI(s)=1$$

$$i(0)=0$$

$$I(s)=\frac{1}{L}\left(\frac{1}{s+R/L}\right)=Y(s)$$ → ラプラス逆変換

インパルス応答

$$i(t)=\frac{1}{L}e^{-\frac{R}{L}t}=h(t)$$

$$I(s)=\frac{E}{R}\frac{1}{\left(s+\frac{1}{RC}\right)\left(1+e^{-\frac{T}{2}s}\right)}=\frac{K}{s+\frac{1}{RC}}+\sum_{n=0}^{\infty}\left(\frac{K_n^-}{s-j\omega_n}+\frac{K_n^+}{s+j\omega_n}\right)$$

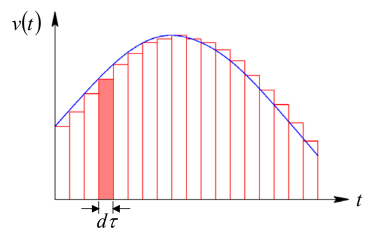
K_n^- は

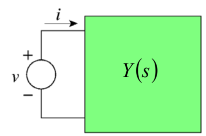
$$K_n^-=(s-j\omega_n)I(s)\Big|_{s=j\omega_n}$$

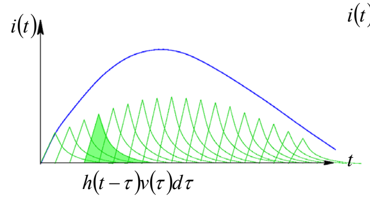
$$=\frac{E}{R}\frac{s-j\omega_n}{\left(s+\frac{1}{RC}\right)\left(1+e^{-\frac{T}{2}s}\right)}\Bigg|_{s=j\omega_n}=\frac{E}{R}\frac{1}{\left(1+e^{-\frac{T}{2}s}\right)+\left(s+\frac{1}{RC}\right)\left(-\frac{T}{2}e^{-\frac{T}{2}s}\right)}\Bigg|_{s=j\omega_n}$$

$$=\frac{E}{R}\frac{1}{\left(j\omega_n+\frac{1}{RC}\right)\left(-\frac{T}{2}e^{-j(2n+1)\pi}\right)}=\frac{E}{R}\frac{1}{\left(j\omega_n+\frac{1}{RC}\right)\frac{T}{2}}$$

インパルス応答を利用した回路の過渡現象解析



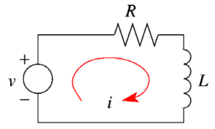




$$i(t)=\mathcal{L}^{-1}\{Y(s)V(s)\}=\mathcal{L}^{-1}\{\mathcal{L}\{h(t)\}V(s)\}$$

$$=\int_0^t h(t-\tau)v(\tau)d\tau$$

$$=\int_0^t h(\tau)v(t-\tau)d\tau$$



$$v(t) = Ee^{-bt}u(t)$$

インパルス応答　$h(t) = \frac{1}{L}e^{-\frac{R}{L}t}$

$$i(t) = \int_0^t h(t-\tau)v(\tau)d\tau = \int_0^t \frac{1}{L}e^{-\frac{R}{L}(t-\tau)}Ee^{-b\tau}d\tau = \frac{Ee^{-\frac{R}{L}t}}{L}\int_0^t e^{-\left(b-\frac{R}{L}\right)\tau}d\tau$$

$$= \frac{Ee^{-\frac{R}{L}t}}{L}\left[\frac{e^{-\left(b-\frac{R}{L}\right)\tau}}{-\left(b-\frac{R}{L}\right)}\right]_0^t = \frac{Ee^{-\frac{R}{L}t}}{L}\frac{1}{b-\frac{R}{L}}\left\{1-e^{-\left(b-\frac{R}{L}\right)t}\right\} = \frac{Ee^{-\frac{R}{L}t}}{bL-R}\left\{1-e^{-\left(b-\frac{R}{L}\right)t}\right\}$$

$$= \frac{E}{bL-R}\left\{e^{-\frac{R}{L}t} - e^{-bt}\right\}$$