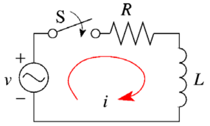
交流回路の過渡現象

・RL直列回路

電源電圧　$v(t)=V_m\sin(\omega t+\theta)$

回路方程式



$$L\frac{di}{dt}+Ri=V_m\sin(\omega t+\theta)$$

$$\Downarrow\quad i=i_t+i_s$$

・定常解　(交流理論より)

$$j\omega LI_s+RI_s=V_e e^{j\theta}\longrightarrow I_s=\frac{V_e e^{j\theta}}{R+j\omega L}=\frac{V_e}{\sqrt{R^2+\omega^2L^2}}e^{j(\theta-\phi)}$$

$$\left(V_e=\frac{V_m}{\sqrt{2}},\quad \phi=\tan^{-1}\frac{\omega L}{R}\right)$$

よって

$$i_s=I_m\sin(\omega t+\theta-\phi)\qquad\left(I_m=\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\right)$$

・過渡解

$$L\frac{di_t}{dt}+Ri_t=0\longrightarrow i_t=Ae^{-t/\tau}\quad\left(\text{時定数}\ \tau=\frac{L}{R}\right)$$

一般解

$$i=i_t+i_s=Ae^{-t/\tau}+I_m\sin(\omega t+\theta-\phi)$$

$\Downarrow$ 初期条件　$t=0$ で $i=0$

$$A+I_m\sin(\theta-\phi)=0\longrightarrow A=-I_m\sin(\theta-\phi)$$

よって

$$i=I_m\sin(\omega t+\theta-\phi)-I_m\sin(\theta-\phi)e^{-t/\tau}$$

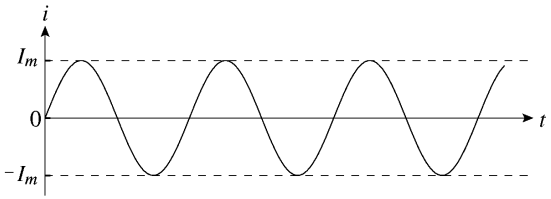
ただし　$I_m=\dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}$,　$\phi=\tan^{-1}\dfrac{\omega L}{R}$,　$\tau=\dfrac{L}{R}$

過渡電流の大きさ $\propto\sin(\theta-\phi)$

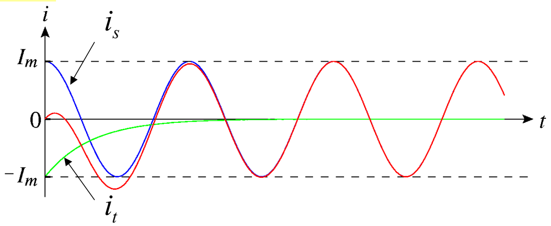
$\theta-\phi=0,\pi\longrightarrow$ 過渡電流 0

$\theta-\phi=\pm\dfrac{\pi}{2}\longrightarrow$ 過渡電流最大

$\theta-\phi=0$



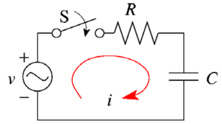
$\theta-\phi=\dfrac{\pi}{2}$



・RC直列回路

電源電圧　$v(t)=V_m\sin(\omega t+\theta)$

回路方程式



$$Ri+\frac{q}{C}=V_m\sin(\omega t+\theta)\xrightarrow{i=\frac{dq}{dt}}R\frac{dq}{dt}+\frac{q}{C}=V_m\sin(\omega t+\theta)$$

$$\Downarrow\quad q=q_t+q_s$$

・定常解

$$j\omega RQ_s+\frac{Q_s}{C}=V_e e^{j\theta}\longrightarrow Q_s=\frac{CV_e e^{j\theta}}{1+j\omega CR}=\frac{CV_e}{\sqrt{1+\omega^2C^2R^2}}e^{j(\theta-\phi)}$$

$$\left(V_e=\frac{V_m}{\sqrt{2}},\quad \phi=\tan^{-1}\omega CR\right)$$

よって

$$q_s=Q_m\sin(\omega t+\theta-\phi)\qquad\left(Q_m=\frac{V_m}{\sqrt{1+\omega^2C^2R^2}}\right)$$

・過渡解

$$R\frac{dq_t}{dt}+\frac{q_t}{C}=0\longrightarrow q_t=Ae^{-t/\tau}\quad(\text{時定数}\ \tau=CR)$$

一般解

$$q = q_t + q_s = Ae^{-t/\tau} + Q_m \sin(\omega t + \theta - \phi)$$

初期条件　$t = 0$ で $q = 0$

$$A + Q_m \sin(\theta - \phi) = 0 \longrightarrow A = -Q_m \sin(\theta - \phi)$$

よって

$$q = Q_m \sin(\omega t + \theta - \phi) - Q_m \sin(\theta - \phi) e^{-t/\tau}$$

ただし　$Q_m = \dfrac{CV_m}{\sqrt{1 + \omega^2 C^2 R^2}}$,　$\phi = \tan^{-1} \omega CR$,　$\tau = CR$

$$i = \frac{dq}{dt} = \omega Q_m \cos(\omega t + \theta - \phi) + \frac{Q_m}{\tau} \sin(\theta - \phi) e^{-t/\tau}$$

$$= I_m \{\cos(\omega t + \theta - \phi) + \cot\phi \sin(\theta - \phi) e^{-t/\tau}\}$$

ただし　$I_m = \dfrac{\omega CV_m}{\sqrt{1 + \omega^2 C^2 R^2}}$,　$\phi = \tan^{-1} \omega CR$,　$\tau = CR$

教科書演習問題問6との関係

$\tilde{\phi} = \tan^{-1} \dfrac{1}{\omega CR}$　と置くと

$$\cot\phi = \frac{1}{\tan\phi} = \frac{1}{\omega CR} = \tan\tilde{\phi}$$

$$\theta - \phi = \theta - \left(\frac{\pi}{2} - \tilde{\phi}\right) = \theta + \tilde{\phi} - \frac{\pi}{2}$$

$$\sin(\theta - \phi) = \sin\left(\theta + \tilde{\phi} - \frac{\pi}{2}\right) = -\cos(\theta + \tilde{\phi})$$

$$\cos(\omega t + \theta - \phi) = \cos\left(\omega t + \theta + \tilde{\phi} - \frac{\pi}{2}\right) = \sin(\omega t + \theta + \tilde{\phi})$$

$$i = I_m \{\sin(\omega t + \theta + \tilde{\phi}) - \tan\tilde{\phi} \cos(\theta + \tilde{\phi}) e^{-t/\tau}\}$$

方形パルス入力に対する応答

*RC*直列回路の過渡現象の一般解

$$q = Ae^{-t/\tau} + CE_s$$

$t = t_0$ で $q = q_0$

$$q = CE_s + (q_0 - CE_s) e^{-(t - t_0)/\tau}$$

$t < 0$

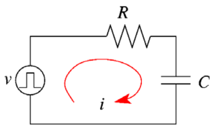
$$q = 0$$

$0 \le t < T$

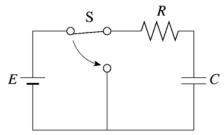
$$q = CE - CEe^{-t/\tau} = CE(1 - e^{-t/\tau})$$

$t > T$

$$q = q(T) e^{-(t-T)/\tau} = CE(1 - e^{-T/\tau}) e^{-(t-T)/\tau}$$



$$v = \begin{cases} E & (0 \le t \le T) \\ 0 & (t < 0,\ t > T) \end{cases}$$



微分回路

・*RL* 直列回路において

$$v_L = L\frac{di}{dt} = \frac{L}{R}\frac{dv_R}{dt} \xrightarrow[v_R \cong v]{\tau = L/R << T} v_L \cong \frac{L}{R}\frac{dv}{dt}$$

（電源電圧の微分に比例）

・*RC* 直列回路において

$$v_R = Ri = R\frac{dq}{dt} = CR\frac{dv_C}{dt} \xrightarrow[v_C \cong v]{\tau = CR << T} v_R \cong CR\frac{dv}{dt}$$

積分回路

・*RC* 直列回路において

$$v_C = \frac{q}{C} = \frac{1}{C}\int idt = \frac{1}{CR}\int v_R dt \xrightarrow[v_R \cong v]{\tau = L/R >> T} v_C = \frac{1}{CR}\int vdt$$

（電源電圧の積分に比例）

・*RL* 直列回路において

$$v_R = Ri = \frac{R}{L}\int v_L dt \xrightarrow[v_L \cong v]{\tau = L/R >> T} v_R \cong \frac{R}{L}\int vdt$$