小テスト

1. (1),(2),(3) のラプラス変換を求めなさい

(1) $f_1(t) = 1$

(2) $f_2(t) = \sin \omega t$

(3) $f_3(t) = e^{j\omega t}$

2. (4),(5) のラプラス逆変換を求めなさい

(1) $F_4(s) = \dfrac{s+2}{s^2+9}$

(2) $F_5(s) = \dfrac{12}{s(s+2)(s+6)}$

解答

(1)

$$F_1(s) = +\int_a^\infty e^{-st}dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{1}{s}$$

(2)

$$\begin{aligned} F_2(s) &= \int_0^\infty \sin \omega t \cdot e^{-st}dt = \frac{1}{j2}\int_0^\infty \left(e^{j\omega t} - e^{-j\omega t}\right)e^{-st}dt = \frac{1}{j2}\int_0^\infty \left\{e^{(-s+j\omega)t} - e^{(-s-j\omega)t}\right\}dt \\ &= \frac{1}{j2}\left[\frac{e^{(-s+j\omega)t}}{-s+j\omega} + \frac{e^{(-s-j\omega)t}}{s+j\omega}\right]_0^\infty = \frac{1}{j2}\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \frac{\omega}{s^2+\omega^2} \end{aligned}$$

(3)

$$F_3(s) = \int_0^\infty e^{j\omega t}e^{-st}dt = \int_0^\infty e^{(-s+j\omega)t}dt = \left[\frac{e^{(-s+j\omega)t}}{-s+j\omega}\right]_0^\infty = \frac{1}{s-j\omega}$$

(4)

$$F_4(s) = \frac{s+2}{s^2+9} = \frac{s+2}{(s+j3)(s-j3)}$$

$$\begin{aligned} f_4(t) &= (s+j3)F_4(s)e^{st}\Big|_{s=-j3} + (s-j3)F_4(s)e^{st}\Big|_{s=j3} \\ &= \left.\frac{(s+2)e^{st}}{s-j3}\right|_{s=-j3} + \left.\frac{(s+2)e^{st}}{s+j3}\right|_{s=j3} = \frac{-j3+2}{-j6}e^{-j3t} + \frac{j3+2}{j6}e^{j3t} \\ &= \frac{1}{2}\left(e^{j3t} + e^{-j3t}\right) - \frac{j}{3}\left(e^{j3t} - e^{-j3t}\right) = \cos 3t + \frac{2}{3}\sin 3t \end{aligned}$$

または，ラプラス変換表を用いて

$$F_4(s) == \frac{s + \frac{2}{3}\cdot 3}{s^2+3^2}$$

より

$$f_4(t) = \cos 3t + \frac{2}{3}\sin 3t$$

(5)

$$\begin{aligned} f_5(t) &= sF_5(s)e^{st}\Big|_{s=0} + (s+2)F_5(s)e^{st}\Big|_{s=-2}\ (s+6)F_5(s)e^{st}\Big|_{s=-6} \\ &= \left.\frac{12e^{st}}{(s+2)(s+6)}\right|_{s=0} + \left.\frac{12e^{st}}{s(s+6)}\right|_{s=-2} + \left.\frac{12e^{st}}{s(s+2)}\right|_{s=-6} \\ &= 1 - \frac{3}{2}e^{-2t} + \frac{1}{2}e^{-6t} \end{aligned}$$