(1)

$$\begin{aligned} F(s) &= \int_0^\infty \sin\omega t\cdot e^{-st}dt = \int_0^\infty \frac{e^{j\omega t}-e^{-j\omega t}}{j2}\cdot e^{-st}dt = \int_0^\infty \frac{e^{-(s-j\omega)t}-e^{-(st+j\omega)t}}{j2}dt \\ &= \frac{1}{j2}\left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)}-\frac{e^{-(s+j\omega)t}}{-(s+j\omega)}\right]_0^\infty = \frac{1}{j2}\left[\frac{1}{s-j\omega}-\frac{1}{s+j\omega}\right] = \frac{1}{j2}\cdot\frac{j2\omega}{s^2+\omega^2} \\ &= \frac{\omega}{s^2+\omega^2} \end{aligned}$$

(2)

$$\begin{aligned} F(s) &= \int_0^\infty \cos\omega t\cdot e^{-st}dt = \int_0^\infty \frac{e^{j\omega t}+e^{-j\omega t}}{2}\cdot e^{-st}dt = \int_0^\infty \frac{e^{-(s-j\omega)t}+e^{-(st+j\omega)t}}{2}dt \\ &= \frac{1}{2}\left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)}+\frac{e^{-(s+j\omega)t}}{-(s+j\omega)}\right]_0^\infty = \frac{1}{2}\left[\frac{1}{s-j\omega}+\frac{1}{s+j\omega}\right] = \frac{1}{2}\cdot\frac{2s}{s^2+\omega^2} \\ &= \frac{s}{s^2+\omega^2} \end{aligned}$$

(3)

$$\begin{aligned} F(s) &= \int_0^\infty e^{-at}\cos\omega t\cdot e^{-st}dt = \int_0^\infty \frac{e^{j\omega t}+e^{-j\omega t}}{2}\cdot e^{-(s+a)t}dt = \int_0^\infty \frac{e^{-((s+a)-j\omega)t}+e^{-((s+a)t+j\omega)t}}{2}dt \\ &= \frac{1}{2}\left[\frac{e^{-((s+a)-j\omega)t}}{-((s+a)-j\omega)}+\frac{e^{-((s+a)+j\omega)t}}{-((s+a)+j\omega)}\right]_0^\infty = \frac{1}{2}\left[\frac{1}{(s+a)-j\omega}+\frac{1}{(s+a)+j\omega}\right] = \frac{1}{2}\cdot\frac{2(s+a)}{(s+a)^2+\omega^2} \\ &= \frac{s+a}{(s+a)^2+\omega^2} \end{aligned}$$

(4)

$$\begin{aligned} f(t) &= (s+a)F(s)e^{st}\Big|_{s=-a} + (s+b)F(s)e^{st}\Big|_{s=-b} = \frac{1}{s+b}e^{st}\bigg|_{s=-a} + \frac{1}{s+a}e^{st}\bigg|_{s=-b} \\ &= \frac{1}{b-a}e^{-at}+\frac{1}{a-b}e^{-bt} = \frac{1}{b-a}\left(e^{-at}-e^{-bt}\right) \end{aligned}$$

(5) 式を整理すると

$$F(s) = \frac{3s+8}{s^2+9} = 3\cdot\frac{s}{s^2+3^2}+\frac{8}{3}\cdot\frac{3}{s^2+3^2}$$

したがって，ラプラス変換表より

$$f(t) = 3\cos 3t + \frac{8}{3}\sin 3t$$

(6) 式を整理すると

$$F(s) = \frac{s+2}{s(s^2+2s+5)} = \frac{K_1}{s}+\frac{K_2(s+1)+K_3}{(s+1)^2+2^2} = \frac{2}{5}\cdot\frac{1}{s}+\frac{1}{5}\cdot\frac{-2(s+1)+3}{(s+1)^2+2^2}$$

したがって，ラプラス変換表より

$$f(t) = \frac{2}{5}u(t) - \frac{1}{5}\left(2\cos 2t - \frac{3}{2}\sin 2t\right)e^{-t}$$