電気回路 II 演習・小テスト (第 13 回)

演習

(1)，(2)，(3) のラプラス変換と (4)，(5) のラプラス逆変換を求めよ．

$$(1)\ f_1(t) = e^{j\omega t}, \quad (2)\ f_2(t) = \cos(\omega t), \quad (3)\ f_3(t) = \begin{cases} t/T & (0 \le t \le T) \\ 0 & (t < 0, t > T) \end{cases}$$

$$(4)\ F_4(s) = \frac{3s+4}{(s+1)(s+2)}, \quad (5)\ F_5(s) = \frac{1}{s^2(s+2)}$$

解答

(1) $f_1(t) = e^{\pm j\omega t}$

$$F_1(s) = \int_0^\infty f_1(t)e^{-st}dt = \int_0^\infty e^{-(s\mp j\omega)t}dt = \left[-\frac{e^{-(s\mp j\omega)t}}{(s\mp j\omega)}\right]_0^\infty = \frac{1}{s\mp j\omega}$$

(2) $f_1(t) = \cos\omega t = \frac{e^{j\omega t}+e^{-j\omega t}}{2}$

$$\begin{aligned} F_2(s) &= \int_0^\infty f_1(t)e^{-st}dt = \frac{1}{2}\int_0^\infty \left(e^{-(s-j\omega)t} + e^{-(s+j\omega)t}\right)dt \quad \left(= \frac{1}{2}\left[-\frac{e^{-(s-j\omega)t}}{(s-j\omega)} - \frac{e^{-(s+j\omega)t}}{(s+j\omega)}\right]_0^\infty\right) \\ &= \frac{1}{2}\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right) = \frac{1}{2}\frac{(s+j\omega)+(s-j\omega)}{s^2+\omega^2} = \frac{s}{s^2+\omega^2} \end{aligned}$$

(3)

$$\begin{aligned} F_3(s) &= \int_0^\infty f_3(t)dt = \int_0^T \frac{t}{T}e^{-st}dt = \left[\frac{t}{T}\frac{e^{-st}}{-s}\right]_0^T + \int_0^T \frac{1}{sT}e^{-st}dt = -\frac{e^{-Ts}}{s} + \left[\frac{e^{-st}}{-s^2T}\right]_0^T \\ &= -\frac{e^{-Ts}}{s} + \frac{1-e^{-Ts}}{Ts^2} \end{aligned}$$

(4) 留数演算から求めると

$$\begin{aligned} f_4(t) &= (s+1)F_4(s)e^{st}\big|_{s=-1} + (s+2)F_4(s)e^{st}\big|_{s=-2} = \left.\frac{3s+4}{s+2}\right|_{s=-1} e^{-t} + \left.\frac{3s+4}{s+1}\right|_{s=-2} e^{-2t} \\ &= e^{-t} + 2e^{-2t} \end{aligned}$$

(5) 留数演算から求めると

$$\begin{aligned} f_5(t) &= \left.\frac{1}{(2-1)!}\frac{d}{ds}\left\{s^2F_5(s)e^{st}\right\}\right|_{s=0} + (s+1)F_5(s)e^{st}\big|_{s=-2} \\ &= \left.\frac{d}{ds}\left\{\frac{1}{s+2}e^{st}\right\}\right|_{s=0} + \left.\frac{1}{s^2}\right|_{s=-2} e^{-2t} \\ &= \left.\left(\frac{t}{s+2} + \frac{-e^{st}}{(s+2)^2}\right)\right|_{s=0} + \frac{e^{-2t}}{4} \\ &= \frac{2t-1+e^{-2t}}{4} \end{aligned}$$