



Korea-Japan
International Workshop of
Nonlinear Partial
Differential Equations

Program and Abstracts

Organizing Committee:

Hideo Kozono (Waseda Univ.)

Takayoshi Ogawa (Tohoku Univ.)

Masaki Kurokiba (Muroran Inst. Tech)

**Korea-Japan International Workshop of
Nonlinear Partial Differential Equations**
—Aspect of Regularity and Asymptotics—

Date : November 17 (Thu) 16:00 ~ 19 (Sat) 21:00

Location : Crystal Hall, Hotel Toya Sun Palace

Address : 〒 049-5721 Toyaonsen 142, Toya-mach kayatagun Hokkaido

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Access : Toya Station of JR-Hokkaido Train, 5min by Taxi.

Program

Nov. 17 (Thu)

16:00–18:00 : Registration

Nov. 18 (Fri)

9:30–9:30 : Opening

9:30–10:20 : Dongho Chae (Chung-ang University)

「Liouville type results in the stationary Euler and the Navier-Stokes equations」

10:30–11:20 : Takayuki Kobayashi (Osaka University)

「Decay property for two-dimensional compressible Navier-Stokes equations」

11:30–12:20 : Erika Ushikoshi (National Yokohama University)

「Hadamard variational formula for the eigenvalue of the Stokes equations
and its application」

12:30-14:00 Lunch Break

14:00–14:50 : Kyungkeun Kang (Yonsei University)

「On regularity of Chemotaxis-fluid equations」

15:00–15:50 : Michiaki Onodera (Tokyo Inst. Tech.)
「Dynamical approach to an overdetermined problem」

15:00–15:50 Coffee Break

16:10–17:00 : Jihoon Lee (Chung-ang Univ)
「Mathematical analysis of a chemotactic model with advection and absorbing reaction」

17:10–18:00 : Hidemitsu Wadade (Kanazawa University)
「On the maximizing problems associated with Trudinger-Moser type inequalities in the whole space」

Nov. 19 (Sat)

10:00–10:50 : Norisuke Ioku (Ehime University)
「Existence and nonexistence of solutions for the heat equation with a super-linear source term」

11:00–11:50 : Hyunseok Kim (Sogang University)
「On L^p -estimates for linear elliptic equations with lower order terms」

12:00-14:00 Lunch Break

14:00–14:50 : Masakazu Kato (Muroran Inst. Tech)
「Asymptotic profile of solutions for the damped wave equation」

15:00–15:50 : Hyong Ohk Bae (Ajou University)
「Short time regularity to unsteady shear thickening incompressible fluids」

15:50–16:10 Coffee Break

16:10–17:00 : Masashi Misawa (Kumamoto University)
「Local regularity theorem for the evolution of p -harmonic maps」

17:10–18:00 : Hi Jun Choe (Yonsei University)
「Regularity of $L^\infty(0, T; L^{3,w}(\mathbb{R}^3))$ solutions to the Navier-Stokes equations」

18:00-18:00 Closing

19:00–21:00 : Conference Dinner

Organizing Committee:

Hideo Kozono (Waseda Univ.)
Takayoshi Ogawa (Tohoku Univ.)
Masaki Kurokiba (Muroran Inst. Tech)

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「Elucidation of uninvestigated problems related to the criticality
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「Research on the blowing up solutions for nonlinear drift-diffusion system
with non-local effects」

Liouville type results in the stationary Euler and the Navier-Stokes equations

Dongho Chae
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Abstract

In this talk we present Liouville type results in the stationary Euler and the Navier-Stokes equations. We first consider the Euler equations with ‘definite signed’ velocity dependent forcing term, which generalizes the usual damping force. In this case we can show that any decaying velocity at infinity is trivial (zero). Next, we discuss the Liouville type properties of the Beltrami flows, which is a stationary solutions of the 3D Euler equations. For such flows it is known previously by Nadirashvili that decay condition $v(x) = o(\frac{1}{|x|})$ as $|x| \rightarrow \infty$, or $v \in L^q(\mathbb{R}^3)$ with $q \in [2, 3]$ implies the triviality, $v = 0$ on \mathbb{R}^3 . This shows that the existence result of a special Beltrami flows by Enciso and Peralta-Salas is sharp. Here we first present an elementary and simple proof of similar result, which was obtained jointly with Constantin. Then, we study recent improvements of those results, deduced jointly with Wolf, which shows that if we assume similar decay rate, but only on the tangential component of the velocity, then the solution becomes trivial. In the second part we present Liouville type results for the stationary Navier-Stokes equations. Here, we present various sufficient conditions, obtained by myself and also with Wolf, which guarantee the triviality of solutions.

Decay property for two-dimensional compressible Navier-Stokes equations

Takayuki Kobayashi

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Abstract

Asymptotic behavior of solutions to the two dimensional compressible Navier-Stokes equation is considered around a given constant equilibrium. The leading part of the solutions is decomposed into two parts, one behaves like diffusion waves and the other one behaves like purely diffusively. Time decay rate in L2-norm of these two parts is same. However, there appear some aspects different from the time decay rate in L2-norm of the solutions, especially in considering time-space L2-norm of the solutions. The results in this talk were obtained in a joint work with M. Misawa (Kumamoto University, Japan) and T. Yanagisawa (Nara Women's University, Japan).

Hadamard variational formula for the eigenvalue of the Stokes equations and its application

Erika Ushikoshi
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Abstract

We consider the domain perturbation problem for the Stokes equations, which describes the motion of the incompressible viscous fluid moving slowly. Hadamard variational formula represents the variation of the eigenvalue or the Green function for some elliptic operator under a certain perturbation of domains. Jimbo-Ushikoshi (2015) succeeded to establish the the formula for the multiple eigenvalue of the Stokes equations with the Dirichlet boundary conditions. In this talk, we discuss the application of that formula to determining the topological type of the domain.

On regularity of Chemotaxis-fluid equations

Kyungkeun Kang
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Abstract

We consider two dimensional chemotaxis equations coupled to the Navier-Stokes equations. We show asymptotics of solutions as well as temporal decays as time tends to infinity under the assumption that the initial mass of biological cell density is sufficiently small. We provide a regularity criterion in a local neighborhood at a point as well. We also study a coupled system consisting of a degenerate porous medium type of Keller-Segel system and Stokes system and we establish the global existence of weak solutions and Holder continuous solutions in dimension three, under the assumption that the power of degeneracy is above a certain number depending on given parameter values.

Dynamical approach to an overdetermined problem

Michiaki Onodera
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Abstract

I will present a recent stability result on an overdetermined problem in potential theory. The main idea is to introduce a flow which appears as a deformation process for a parametrized “stationary” problem. Spectral analysis for the linearized flow yields a detailed stability estimate for solutions to the overdetermined problem.

Mathematical analysis of a chemotactic model with advection and absorbing reaction

Jihoon Lee
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Abstract

We consider a parabolic-parabolic equations which describes some chemotactic model with advection and absorbing reaction, which read as :

$$\begin{cases} \partial_t n + u \cdot \nabla n - \Delta n = -\chi \nabla \cdot (n \nabla c) - \epsilon n^q, \\ \partial_t c + u \cdot \nabla c - \Delta c = n - c, \end{cases} \quad \text{in } \mathbb{R}^d \times (0, T), \quad (1)$$

where $c \geq 0$, $n \geq 0$, and u denote the chemical concentration, cell density and a given divergence free ($\nabla \cdot u = 0$) velocity of sea fluid, respectively. In the above, χ and ϵ are positive constants.

We establish the local and global well-posedness of regular solutions for the model. We also prove that the total mass of the cell density asymptotically approaches a strictly positive constant, provided that efficiency of absorbing reaction is strong enough. This talk is based on the joint work with Jaewook Ahn(Kyushu U.), Kyunkeun Kang(Yonsei U.) and Junha Kim(Chung-Ang U.).

On the maximizing problems associated with Trudinger-Moser type inequalities in the whole space

Hidemitsu Wadade
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Abstract

Trudinger-Moser inequality was originally proved by Trudinger and Moser, which guarantees the exponential integrability for functions in the critical Sobolev spaces. Trudinger-Moser inequality has been extended into several forms, from bounded domains to unbounded domains for instance. Among others, two types of Trudinger-Moser type inequalities in the whole space are known, one has a scaling invariant property and another doesn't have. In this talk, we consider the maximizing problems associated with those two types of Trudinger-Moser's inequalities, and clarify the difference between the attainability for the corresponding best constants, which is affected by the scaling properties and the ways to adopt the normalization norms. These results are joint works with Prof. Ishiwata in Osaka University.

Existence and Nonexistence of Solutions for the Heat Equation with a Superlinear Source Term

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Abstract

We consider the heat equation with a superlinear source term:

$$\begin{cases} \partial_t u = \Delta u + f(u) & \text{in } \mathbb{R}^N \times (0, T), \\ u(x, 0) = u_0(x) \geq 0 & \text{in } \mathbb{R}^N, \end{cases} \quad (\text{P})$$

where $\partial_t = \partial/\partial t$, $N \geq 1$, $T > 0$, u_0 is a nonnegative initial function and f is a positive monotonically increasing function in $(0, \infty)$ with superlinear growth. We consider the case $u_0 \notin L^\infty(\mathbb{R}^N)$ and investigate local in time existence and nonexistence of solutions for problem (P) without any concrete assumption on the growth rate of f . In particular, we reveal the threshold integrability of u_0 to classify existence and nonexistence of solutions for problem (P). This is a joint work with Professor Yohei Fujishima (Shizuoka University).

References

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- [2] Y. Fujishima and N. Ioku, *Existence and nonexistence of solutions for the heat equation with a superlinear source term*, preprint.
- [3] J. Ginibre and G. Velo, *The Cauchy problem in local spaces for the complex Ginzburg-Landau equation II. Contraction methods*, Commun. Math. Phys. **187** (1997), 45–79.
- [4] F. B. Weissler, *Local existence and nonexistence for semilinear parabolic equations in L^p* , Indiana Univ. Math. J. **29** (1980), 79–102.

On L^p -estimates for linear elliptic equations with lower order terms

Hyunseok Kim
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Abstract

On L^p -estimates for linear elliptic equations with lower order terms Abstract: We consider the Dirichlet problem for second-order linear elliptic equations with the first-order term given by a singular vector field u . $W^{1,p}$ -estimates for weak solutions are derived for $u \in L^n$, where $n \geq 3$ is the spatial dimension. Resolvent estimates in $W^{-1,p}$ or L^p are also provided. Finally, we discuss the case of more singular u which is motivated by a uniqueness problem for the stationary Navier-Stokes equations in exterior domains.

Asymptotic profile of solutions for the damped wave equation

Masakazu Kato
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Abstract

We consider the large time behavior of solutions to the initial value problem for the damped wave equations with nonlinear convection in one-dimensional whole space. The damped wave equation with a nonlinear convection term is derived from a nonlinear hyperbolic relaxation system by using the Chapman-Enskog expansion. The system is one of important mathematical models which describe physical phenomena. For example, non-equilibrium gas dynamics, magnetohydrodynamics, viscoelasticity and flood flow with friction are expressed by nonlinear hyperbolic relaxation systems. In Ueda-Kawashima, the authors showed that the solutions tend to a self-similar solution of the Burgers equation. However, they did not mention that their decay estimate is optimal or not. Under this situation, the aim of this talk is to find out the sharp decay estimate by studying the second asymptotic profile of solutions. The explicit representation formula and the decay estimates of the solution for the linearized equation including the lower order term play a crucial role in our analysis. We find that the decay rate heavily depends on the structure of the nonlinearity. We remark that the similar estimate was obtained for Burgers type equations and the situation for the generalized Burgers equations is slightly different from the damped wave equations.

SHORT TIME REGULARITY TO UNSTEADY SHEAR THICKENING INCOMPRESSIBLE FLUIDS

HYEONG-OHK BAE AND JÖRG WOLF

Abstract

We address the existence of strong solutions to a system of equations of motion of an incompressible non-Newtonian fluid. Our aim is to prove the short-time existence of strong solutions for the case of shear thickening viscosity, which corresponds to the power law $\nu(\mathbf{D}) = |\mathbf{D}|^{q-2}$ ($2 < q < +\infty$). In particular, we find that global strong solutions exist whenever $q > 2.23\dots$. The results are obtained by flattening the boundary and by using the difference quotient method. Near the boundary, we use weighted estimates in the normal direction.

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Local regularity theorem for the evolution of p-harmonic maps

Masaki Misawa
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Abstract

We study a geometric analysis for the evolutionary p-Laplace operator and show a local regularity theorem for the evolution of p-harmonic maps, called p-harmonic map heat flows. Our main result is to establish a criterion for a uniform local regularity estimate for regular p-harmonic map heat flows, devising some new monotonicity type formulas of a local scaled energy. The regularity criterion obtained is almost optimal, comparing with that of the corresponding stationary case. As application we show a compactness of regular p-harmonic map heat flows with energy bound.

Regularity of $L^\infty(0, T; L^{(3,w)}(\mathbb{R}^3))$ solutions to the Navier-Stokes equations

Hi Jun Choe
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Abstract

We prove that if a weak solution u to the Cauchy problem of the incompressible Navier-Stokes equations satisfies the condition $L^\infty(0, T; L^{(3,w)}(R^3))$, then it is smooth in $R^3 \times (0, T)$. Furthermore, there are at most a finite number of blowup points at the final time $t = T$. Here, $L^{(3,w)}(R^3)$ denotes the usual weak Lebesgue space.

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